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Credit risk models in risk-neutral valuation of options and guarantees

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Until recently, a lot of companies limited stochastic valuation of options and guarantees to interest rate risk and equity risk. In reality, credit risk is an important risk factor which has often been neglected. The volatility of credit spreads can be an important contributor of the cost of options and guarantees, particularly for products with guaranteed surrender values. In many countries, regulators used to look at this issue quite liberally, but this attitude has started to change, which has led to an increased focus on this risk.

Executive summary

We took a closer look at the way joint interest rate and credit risk for risk-neutral valuation are typically offered by providers of Economic Scenario Generators ("ESG's"). One thing which drew our attention is the fact that typically correlation (dependence) between interest rate risk and credit risk is not taken into account in the most common ESG solutions available in the market. A possible reason for this might be that interest rate models have become too complex to handle the dependencies with other risk factors.

However, analysis of historical time series of interest rates and credit spreads clearly shows that at least in some periods, correlation between interest rates and credit can be significant. In this paper we have examined the extent to which neglecting correlation can have impact on valuation results.

We considered the case in which joint interest rate and credit risk dynamics is described by affine processes, and in particular we modelled the case when interest rates follow a very popular G2++ model, while credit risk is represented by the so called LMN model. This allowed us to make numerical case studies comparing three models:

1. A common advanced interest rate model offered by ESG providers, the so-called DDSVLMM model with credit risk but without dependence
2. G2++ model for interest rates with credit risk but without dependence
3. G2++ model for interest rates with credit risk and modelling dependence

In the case studies, we demonstrate that the difference related to the choice of interest rate model can sometimes be much less significant than the impact of dependence modelling. We conclude that overfocusing on developing a sophisticated interest rate model at the cost of coherency with other risk factors (in this case credit) can lead to potentially serious valuation bias.

The modelling background

Financial modelling of credit risk can be captured within risk-neutral ESGs.

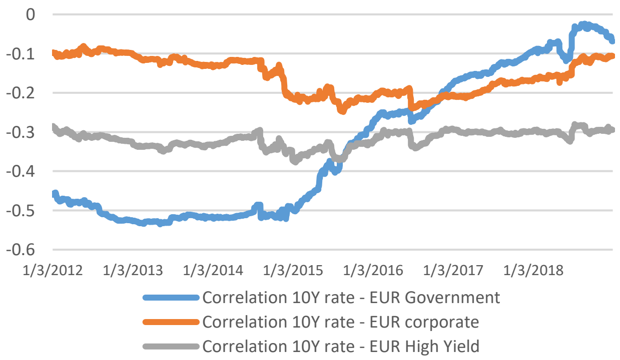
However, most ESG providers take a simplified approach to dealing with credit risk. One of the main simplifications is related to an implicit assumption of independence between credit risk and interest rates. This assumption facilitates modelling, as it allows for combining different types of interest rate models with different types of credit risk model within ESGs, while for many of those combinations explicit modelling of dependence would be difficult. In reality, there is strong evidence of dependence between interest rates and credit risk, and neglecting this fact might be distortive.

In Figure 1, we show three-year correlations between 10-year swap rates and credit spreads calculated over the period between 1 January 2012 and 31 December 2018, considering three different benchmarks for credit spreads: EUR government bonds, EUR corporate bonds and EUR high-yield bonds.

This data suggests that dependency between interest rates and credit risk may be an important factor in practice. We note that:

- The correlation between 10-year interest rates and EUR government bond spreads appears to depend on economic conditions (varying between around -50% and 0% during the period), which suggests that it should be carefully considered at each valuation date.
- The correlation between 10-year interest rates and EUR corporate bond spreads was generally fairly stable (at around -10%), but in some periods decreased to around -25%.
- The correlation between 10-year interest rates and EUR high-yield bond spreads was the most stable, varying during the period between around -30% and -35%.

FIGURE 1: CORRELATION 10-YEAR INTEREST RATE – CREDIT SPREADS



The impact of dependency between interest rates and credit risk was the driving motive for this piece of research.

In this article, we describe how to deal with dependence between interest rates and credit, which is often neglected in ESG models. A possible reason for this is that interest rate models offered by ESG providers are often very complex, making it difficult to model the interaction with other risks. We demonstrate how it is possible to integrate credit models with interest rate models which belong to the family of so-called affine processes. We modelled two special cases of such constructions: G2++ model for interest rates (currently one of the most widely used interest rate models, see [2]) combined with credit risk modelled with an approach introduced by Longstaff, Mithal, Neis (2005) (LMN model, see [4]) in which the dynamics of credit risk is driven by a Cox-Ingersoll-Ross (CIR) process. We have also shown the effect of using as an interest rate model the Displaced Diffusion and Stochastic Volatility Libor Market Model or DDSVLMM, one of the most sophisticated models available in the market, with an assumption of independence with credit risk.

For modelling dependence between interest rates and credit risk, we advocate to capture those risks with a common structure of so-called affine processes, although we should emphasize that it is not always straightforward. Even for our base model, G2++ model for interest rates (which is affine) and the LMN model for credit risk (which is also affine), the combination of correlated processes is not affine and in such case affine approximations need to be used, as in [1]. We describe our approach in details in [3].

We have performed case studies: valuation of a regular premium participating product with different sets of economic scenarios in order to illustrate the impact of different modelling choices — in particular, the choice of interest rate model and modelling of dependence between interest rates and credit risk. In our illustration, the impact of dependency modelling was more significant than the impact of the choice of interest rate model. Whilst we cannot conclude that this must be a general rule, the examples do demonstrate that abandoning dependence modelling in order to be able to apply more sophisticated interest rate models might not always be justifiable. Independence of interest rates and credit is in fact quite a strong assumption, and it should be at least preceded by an impact study, as for any other assumption applied by a financial economist or an actuary.

Case studies

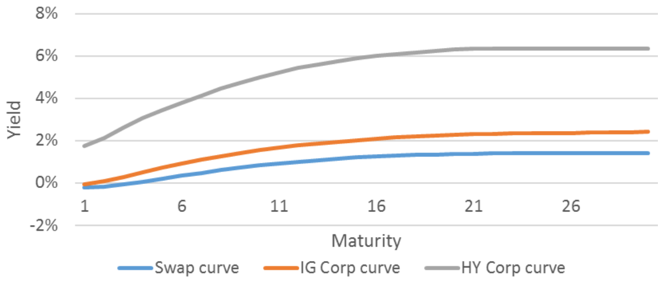
We performed valuations of a 30-year regular premium participating product for different sets of economic scenarios. One of the main purposes of these numerical illustrations was to examine the possible impact of modelling dependence between interest rates and credit risk, and to compare this impact to the impact of other important modelling choices, in particular the choice of a simpler or much more sophisticated interest rate model.

We applied valuations in the following steps:

1. DDSVLMM for interest rates with LMN model for credit, with an assumption of independence of interest rates and credit risk
2. G2++ for interest rates with LMN model for credit, with an assumption of independence of interest rates and credit risk
3. G2++ for interest rates with LMN model for credit, without an assumption of independence of interest rates and credit risk

In our analysis, we were particularly interested in answering the question of what has higher impact: moving between 1 and 2 (sophistication of the interest rate model) or between 2 and 3 (dependence modelling). We note that this is a clear trade-off, as it would be very difficult (if not impossible) to consider dependency modelling for a very sophisticated interest rate model like DDSVLMM (which is not an affine process). In our view, once users decide to use such a model for interest rates, in consequence credit risk has to stay independent for interest rates, which as we have shown is probably not a realistic assumption.

FIGURE 2: YIELD CURVES USED FOR CALIBRATION



MODELLED PRODUCT AND ALM MODEL SETUP

On the liability side, we defined a 30-year regular premium participating product with a minimum guarantee of 0.5% and profit sharing based on book value investment returns as seen in a number of continental European countries. The initial mathematical reserve modelled was equal to EUR 621 million.

We have assumed the following asset mix:

- Italian government bonds with initial market value of EUR 309 million
- Investment-grade corporate bonds with initial market value of EUR 232 million
- High-yield corporate bonds with initial market value of EUR 234 million
- Each year the portfolio is rebalanced to the following proportion:
  - Italian government bonds – 40%
  - Investment-grade corporate bonds – 30%
  - High-yield corporate bonds – 30%
- All purchased bonds have duration of five years

For corporate bonds, we applied the credit risk model, while Italian government bonds credit spreads were modelled on a deterministic basis.

ECONOMIC SCENARIOS

We performed standard calibration for DDSVLMM and G2++ to the risk-free rates and swaption volatility surface for 31 December 2018. In this calibration, we considered swaptions with different maturities, tenors and strikes.

We calibrated interest rate and credit risk model to the following curves

- Swap curve for risk-free rates
- Investment-grade curve based on yield curve for rating A
- High-yield curve based on yield curve for rating BB

The three curves are presented in Figure 2.

The assumed correlation between interest rates and both investment grade and high-yield logarithmic spreads were consistent across different maturities and equal to:

- Investment grade vs. interest rates: about -0.05
- High-yield vs. interest rates: about -0.3

NUMERICAL ILLUSTRATIONS

In the table below, we summarize the valuation results.

By means of our illustrative results we demonstrated that in some cases introducing dependence between credit risk and interest rates can impact the Time Value of Financial Options and Guarantees ("TVFOG") more than applying a more sophisticated interest rate model. Affine models form a general coherent framework which allows them to deal with dependency between interest rates and credit risk, and at the same time they are very tractable as term structure models.

We emphasize that this type of impact will not necessarily always apply and different cases may very likely show different effects. For example, we could expect that for companies with low ratios of high-yield corporate bonds, the impact of dependency modelling on TVFOG would be limited as at the end of 2018 due to the low correlation. Note, however, that two years earlier, at the end of 2016, this correlation was not as low as at the end of 2018, so this low correlation cannot be considered as a rule.

The impact of dependency modelling on time value of financial options and guarantees might also depend on other factors, such as the liability profile, asset mix, assumed reinvestment strategy, level of correlations and other model assumptions. Nevertheless, we can say in conclusion that ignoring dependency between interest rates and credit risk is potentially quite a strong assumption and should not be made without careful consideration. The need to take account of such potential dependencies is a factor which should be considered when choosing the interest rate model to be used.

Case	Stoch BEL	CE BEL	TVFOG	% CE BEL	Delta's
No credit risk	613.3	610.3	3.0	0.49%	
LMM+ for interest rates, CIR for credit, assumption of independence	622.0	610.3	11.7	1.92%	+1.43%
G2++ for interest rates, CIR for credit, assumption of independence	621.7	610.3	11.4	1.87%	- 0.05%
G2++ for interest rates, CIR for credit, dependence modelled	619.8	610.3	9.5	1.56%	-0.31%

Literature

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[4] Longstaff, F. A., Mithal, S., Neis, E. (2005). Corporate yield spreads: Default risk or liquidity? New evidence from the credit default swap market. *The Journal of Finance*, 60(5), 2213-2253.

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