Diversification of longevity and mortality risk

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INTRODUCTION

One sure thing is that the future will be different. Actuaries use sophisticated techniques involving both science and art to develop projections of future contingencies and their financial ramifications. But, as good as they are, actuaries lack precognition, and there will always be some difference between the future experience and the modeled results. Hence, prudent actuaries will examine results with margins applied to their assumptions to cover potential deviations of results. But what margins are appropriate? While a margin may feel tangible (e.g., 10%), it may actually be arbitrary. What is the probability that the margin will be exceeded? And how do the margins on different product types interact?

This case study will explore those questions using a simple combination of life insurance and payout annuity products by applying stochastic projections of future mortality rates. We will compare percentile values from the stochastic projections to results using deterministic projections with margins. We will also demonstrate the relative diversification benefit of the longevity exposure from the annuity product along with the mortality exposure of the life insurance product.

CORRELATION OF MORTALITY IMPROVEMENT

It is a generally held belief among actuaries that the risk of adverse mortality experience on life insurance may be offset by the longevity risk associated with annuity benefits. That is, actuaries believe that an unexpected rise in mortality rates among insureds that increases the amount of death claims paid will be offset, in whole or in part, by a reduction in reserves held for future annuity payouts to reflect the reduced number of survivors. The converse is also commonly believed (i.e., a decrease in mortality rates experienced will result in losses on payout annuities to be offset by reduced life insurance death claims).

This may be true, but the question we address in this case study is, to what extent?

We analyzed the pattern of mortality improvement based on U.S. population mortality experience from 1960 to 2010.[1] Consider the relationship of historical mortality improvement at two ages: age 35, a plausible age for an in-force term life insurance policy, and age 75, a reasonable age to be receiving a payout annuity. Looking back over the 50-year period, the correlation between the two sequences of annual mortality improvement is 1.8% for males and 25.4% for females.

FIGURE 1: ANNUAL MORTALITY IMPROVEMENT - MALE U.S. POPULATION 1960-2010

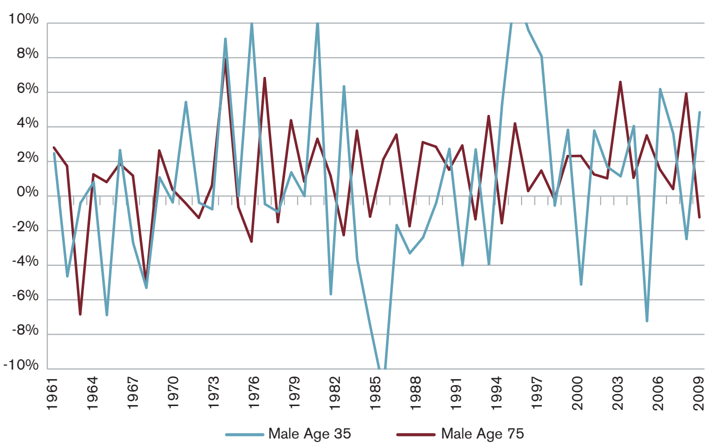
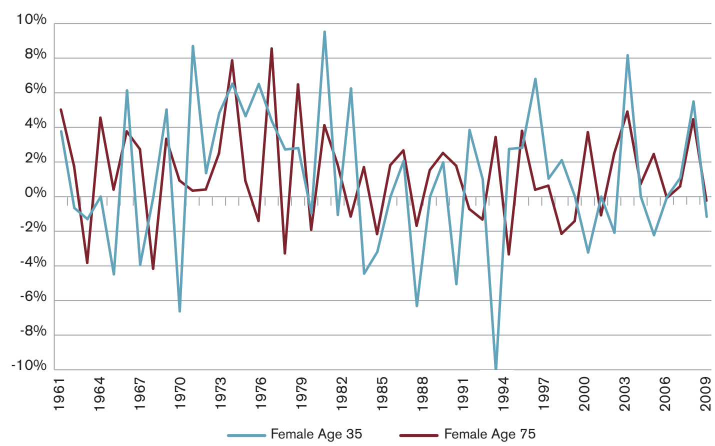


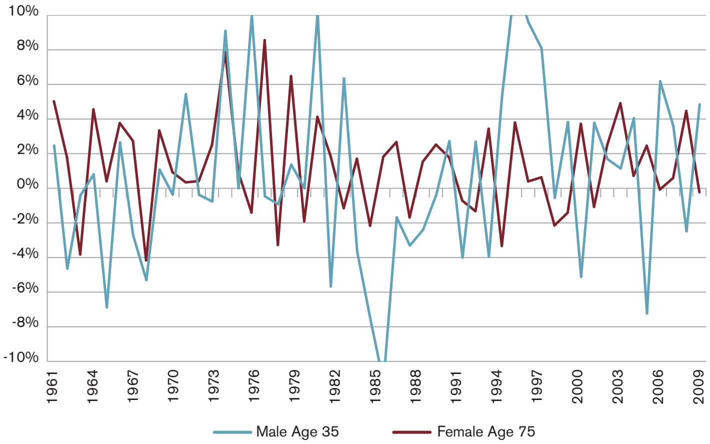
FIGURE 2: ANNUAL MORTALITY IMPROVEMENT - FEMALE U.S. POPULATION 1960-2010



1 Human Mortality Database, accessed January 25, 2016, at http://www.mortality.org.

An interesting comparison of male age 35 and female age 75 might also be illuminating, assuming a plausible distribution of benefits with men having more insurance coverage and women surviving longer to receive annuity payouts. In this case, the correlation over the period 1960 to 2010 is still just 8.4%. See Figure 3.

FIGURE 3: ANNUAL MORTALITY IMPROVEMENT - MALE & FEMALE U.S. POPULATION 1960-2010



These results suggest that there is not that much correlation. While the annual volatility impacts year-to-year cash-flow patterns and should be reflected, it doesn't tell the entire story. Let's dive a little deeper.

One possible explanation is that the lack of correlation may be attributable to one-year variations in results for the single ages being considered. To reduce this statistical "noise," we examine five-year moving averages of annual volatility over the period, applied to the annual improvement averaged over the 10-year age groups centered on the selected age (ages 30-39 and 70-79).

The resulting correlation is 26.3% for males and 60.1% for females. While we begin to see more correlation using this technique, the exposure period has significant impact on the results. The last 25 years (1985 to 2010) were much less correlated than the first 15 years. If we were to limit our comparison to the last 25 years, we find that the correlation drops to -38.1% and 4.7% for males and females, respectively.[2] See Figures 4 and 5.

Therefore, any financial risk model should not presume that the presence of annuity benefits is a near perfect offset for adverse deviations on life insurance risks. This leads us to ask just how companies might evaluate the margins inherent in their economic surplus on their life insurance and annuity blocks of business.

FIGURE 4: ANNUAL MORTALITY IMPROVEMENT - MALE U.S. POPULATION FIVE-YEAR MOVING AVERAGE 1960-2010

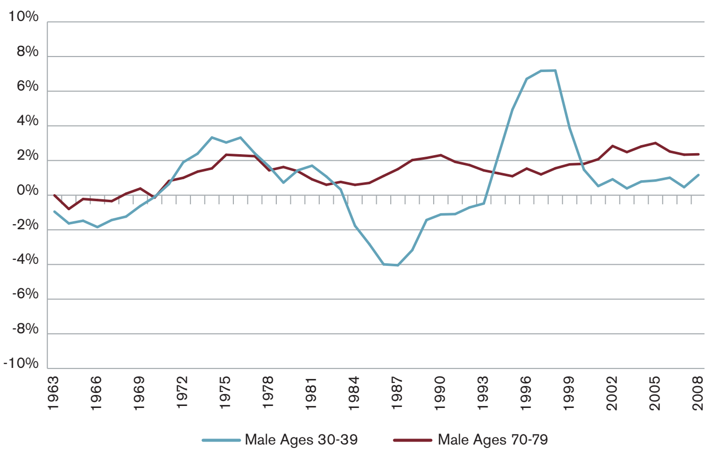
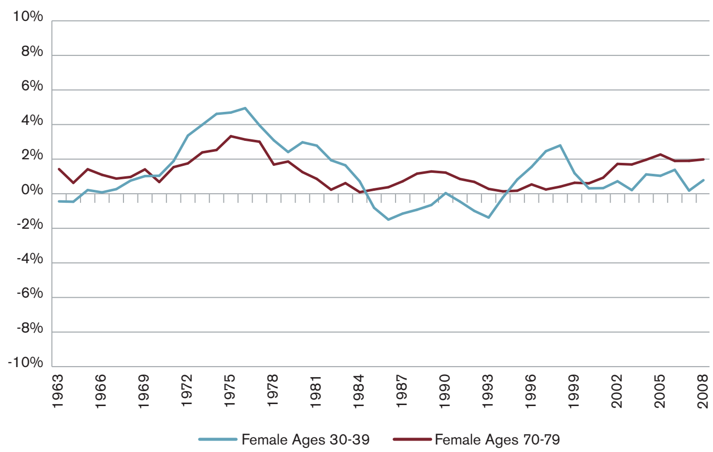


FIGURE 5: ANNUAL MORTALITY IMPROVEMENT - FEMALE U.S. POPULATION FIVE-YEAR MOVING AVERAGE 1960-2010



2 As an aside, it is worth noting that the grouping of ages and applying a moving average has the effect of reducing the annual volatility as seen in Figures 4 and 5, and the standard deviations in mortality improvement rates as well:

STANDARD DEVIATION OF ANNUAL MORTALITY IMPROVEMENT 1960-2010

	AGE 35	AGES 30-39 AND MOVING AVERAGE	AGE 75	AGES 70-79 AND MOVING AVERAGE
Male	5.20%	2.60%	2.82%	0.94%
Female	4.35%	1.70%	2.94%	0.83%

SAMPLE PRODUCTS USED IN MODELING

This case study will be based on a portfolio consisting of two simple life insurance products, a level-premium term life insurance policy and an immediate payout annuity. The following list summarizes key characteristics of the products.

1. Level premium term life insurance
   - $1 billion total face
   - ~$3.3 million first-year premium
   - 10- / 20- / 30-year level term (20% / 50% / 30%)
   - Underwriting classes: PNS / NS / PSM / SM (50% / 30% / 15% / 5%)
   - Male (75%) / Female (25%)
   - 10-year term (40% age 35 / 60% age 55)
   - 20-year term (40% age 35 / 60% age 55)
   - 30-year term (65% age 35 / 35% age 50)
2. Immediate payout annuity
   - Life only – No period certain
   - Males ages 65, 75, and 85
   - $0.5 million annualized payments ($212.5K / $162.5K / $125K at ages 65, 75, and 85, respectively)
   - $5.4 million single premium ($3.0M / $1.7M / $0.7M at ages 65, 75, and 85, respectively)

These hypothetical products were designed to produce expected statutory profits after reflecting initial acquisition costs, reserve strain, and the cost of capital ("distributable earnings") as shown in Figure 6:

FIGURE 6: BASELINE ("BEST ESTIMATE")

DISCOUNT RATE	PRESENT VALUE OF DISTRIBUTABLE EARNINGS		
	COMBINED	ANNUITY	TERM
4%	$4,022,238	$145,925	$3,876,313
8%	$525,466	$21,033	$504,433
12%	($1,100,518)	($47,894)	($1,052,624)
IRR	9.00%	9.00%	9.00%

The choice of a pure term life insurance product (without cash value build-up) and a simple life-contingent single-premium immediate annuity (SPIA) without any other benefits or features reduces the effects of extraneous factors, allowing us to analyze the diversification effect of changes in mortality on different products. However, even these simple products are subject to other factors that will affect risk and profitability. For example, the term life insurance product is exposed to lapse risk, which is not applicable to the SPIA product. There are differing durations for the liabilities, which create different levels of asset exposure. While these product choices were selected to mitigate exposure to these other risks, the insurer's actual portfolios of business will have a range of varying risk exposures that should be all considered in aggregate.

For this case study, we purposely chose a relatively small annuity portfolio compared with the life insurance portfolio. Our intention was for our hypothetical aggregate portfolio to be more aligned with the net risk exposure of the majority of life insurers today. That is, most life insurers currently have more mortality risk exposure relative to their longevity risk exposure. However, it is likely that this relationship will change over time to reflect changing demographics. As such, later in this case study, we demonstrate the implications of changes in these proportions, with increased longevity risk exposure.

ARE DETERMINISTIC MARGINS APPROPRIATE IN CONTEXT OF THE OVERALL RISK LEVEL?

Insurance companies may apply fixed margins to their best estimate mortality assumptions to provide a cushion against adverse deviation when pricing. When examining the financial strength of the company (e.g., economic capital), insurers may test results with even more conservative margins. How does the insurer know if the margin is reasonable? While a margin may feel tangible, it may actually be arbitrary.

Deterministic margins may be expressed as explicit scalar adjustments or may be embedded in the mortality table itself. For this analysis, let us consider explicit margins applied to the mortality table and the annual mortality improvement:

a. 5% of annual mortality rates
b. 0.50% adjustment to annual mortality improvement

That is, for the term life insurance, the adjusted mortality will be:

a. **105**% of best estimate annual mortality rates
b. Best estimate annual mortality improvement rates **reduced** 0.50%

For the payout annuity, the adjusted mortality will be:

a. **95**% of best estimate annual mortality rates
b. Best estimate annual mortality improvement rates **increased** 0.50%

Figure 7 illustrates results with the deterministic margin and then compares the results with the baseline (which excludes the deterministic margin).

FIGURE 7: PRESENT VALUE OF DISTRIBUTABLE EARNINGS - DETERMINISTIC MARGIN

DISCOUNT RATE	PRESENT VALUE OF DISTRIBUTABLE EARNINGS			COMBINED COST OF MARGIN VS. BASELINE
	COMBINED	ANNUITY	TERM	
4%	$2,093,713	$31,751	$2,061,961	$1,928,525
8%	($537,499)	($46,545)	($490,955)	$1,062,965
12%	($1,745,275)	($92,432)	($1,652,843)	$644,757
IRR	6.89%	5.36%	6.95%	2.11%

We have developed the values shown in Figure 7 to be comparable with those in Figure 6. We recognize that real-world pricing objectives would be generally higher than the IRRs shown here.

While it is easy to simply overlay a deterministic margin on to all products, the analysis above does not consider the potential diversification benefits. The assumptions for this analysis reflect higher mortality for the life business and lower mortality for the annuity business, without consideration of the diversification benefit of changing mortality rates. While this may be common practice, it may lead to an overestimation of the aggregate risk, or at least it may call the appropriateness of deterministic margins into question.

Many insurance companies might protest that it is difficult to examine the diversification benefit. Historically, most companies had minimal longevity risk exposure, which allowed them the luxury of not having to calculate the diversification effect directly. However, given shifts in demographics and the growing importance of longevity risk products, detailed analysis of the diversification benefits of life and annuity business is becoming increasingly important. In our roles as consultants to the insurance industry, we find that companies with which we discuss this topic are becoming increasingly interested in understanding the benefits. Further, we have noticed that companies that have a more diverse portfolio of business are a little further along in understanding the risk diversification than those that do not.

Below we demonstrate a method for understanding these diversification benefits for life and annuity products.

INTRODUCING VOLATILITY TO MORTALITY ASSUMPTIONS

As shown above, we have seen periods of changes in mortality where rates moved in a reasonably similar pattern, and other periods where they did not move in similar patterns. We can use stochastic analysis to simulate these patterns to examine possible changes in future mortality rates. Instead of using deterministic margins, in the following analysis we stochastically generated mortality rates based on historical levels of mortality rate volatility and correlation to examine the potential change in the profitability of the combined life and annuity business.

We used REVEAL,[3] a Milliman software platform that is used to examine volatility in mortality rates, to quantify the potential ranges of profitability for the hypothetical life and annuity blocks of business. Specifically, we generated 1,000 stochastic scenarios to model the volatility around future mortality improvement rates (trend risk), the base mortality table itself (basis risk), and the potential variation of life insurance selection period (long-term underwriting risk).

A. Trend risk

As shown above, historical levels of mortality improvement have not followed smooth and predictable trends. The pattern of mortality improvement is important because, when determining the emergence of profits, we are less concerned about modeling average mortality improvement statistics and more concerned with resulting year-to-year cash flows. While we acknowledge the work that has been done developing assumptions for expected mortality improvement, an inspection of historical experience reveals discontinuities and irregular fluctuations. The present value of projected benefit payments is affected by this volatility in the rates of mortality improvement.

Mortality improvement may be perceived as a combination of long-term waves with lingering effects over multiple years, and random annual fluctuations around these long-term waves. To project future mortality improvement volatility, we utilize historical levels of improvement observed in general U.S. population data over the period from 1970 to 2010,[4] specifically focused on three factors:

- Long-term mortality improvement trends –The long-term movements may be the result of various factors, including events in medical practice, medical research, societal changes, economic shifts, political activities, and environmental changes.

  For this analysis, our projected long-term mortality improvement volatility was assumed to cover 10-year periods, based on volatility parameters determined from historical levels of mortality improvement volatility over consecutive 10-year intervals.

- Short-term (annual) mortality improvement volatility – Concurrent with long-term mortality improvement trends, historical mortality improvement rates fluctuate from year to year. These fluctuations can be attributed to multiple factors, including extreme weather conditions, new disease strains, or even variations in reporting.

  This analysis used projected annual mortality improvement volatility, based on volatility parameters captured from historical levels of annual mortality improvement volatility, while ensuring our long-term mortality improvement volatility target assumptions are also met.

- Correlation in mortality improvement trend volatility –We analyzed the correlation of annual and long-term mortality improvement

3 REVEAL is a system developed to analyze mortality and longevity risk. REVEAL generates stochastic projections of liabilities with volatile assumptions (i.e., baseline mortality, mortality improvement, extreme mortality, and longevity events). Find more information about REVEAL at http://www.milliman.com/Solutions/Products/REVEAL/.

4 While the graphs in Figures 1-5 show a longer period of history, we chose to use the experience from a more recent period for setting this assumption.

across ages and genders. The random values used to generate stochastic future improvement reflected the observed correlation.

The excesses and/or shortfalls of population mortality improvement[5] over the historical averages from each scenario in the stochastic projections were applied to the expected mortality improvement rates for the insured and annuitant lives in both of the pricing models.

B. Basis risk

The assumed mortality is based on standard industry tables. While (by definition) industry experience is consistent with this assumption, the business placed with any given insurer may reflect different characteristics from the average characteristics at companies that contributed to the development of these industry tables.

For example, the risks associated with annuitant lives may vary by occupation, size of policy, or region. For life insurance, volatility is introduced during the underwriting process by the assignment of each life to discrete underwriting classes, each of which may cover a range of expected mortality.

Therefore, we modeled the risk that the specific block may have experience that is proportional to the baseline mortality assumption, but not necessarily equal to it. Specifically, for this analysis we assumed that, for each scenario, a single scalar is applied to the respective baseline mortality rates in all years for all ages with a normal distribution around an expected value of 100% and a standard deviation of 5.00%.[6] However, unlike the trend risk, where the term and annuity mortality were simulated in each scenario, reflecting historical levels of correlation between ages and gender, the random basis risk is generated separately and independently for term life and payout annuities within each scenario, reflecting the fact that the insurer likely has different sales and underwriting approaches to these two blocks of business.

While we used a hypothetical assumption for this case study, in practice the magnitude of the volatility parameter for analyzing this risk will depend on the degree of certainty the actuary has about the starting baseline assumption. For example, if the actuary has a lot of experience that exhibits consistency, it would be reasonable to assume only a small amount of basis risk volatility. However, if the actuary has limited mortality experience for developing the assumption (e.g., pricing a pension buyout annuity for a pension plan with limited experience), it may be more reasonable to reflect a higher degree of uncertainty in the basis risk parameter.

C. Long-term underwriting risk

Underwriting places individual lives in a class that will have better experience than those who have not been selected. Furthermore, it has been demonstrated that this effect wears off as the time passes from underwriting. The use of select and ultimate mortality has expanded from five years in the not-so-distant past to 25 years in recent industry tables. This leads us to speculate whether selection ever disappears. Specifically, does either the "preferred" status or a substandard rating wear off over time, so that all lives ultimately grade toward standard? Emerging experience suggests that the underwriting impact may wear off to some extent over time, but the degree and timing is not fully known.[7]

To address the uncertainty with long-term underwriting risk, we examine 1) the uncertainty around the length of the initial selection period, 2) the period over which the preferred or substandard mortality rating takes to wear off, and 3) the ultimate level of mortality after the completion of the wearing off. For this case study, the approach we used to model the long-term underwriting risk consists of three components applied in addition to the parameters describing the basis risk:

1. Initial selection period in years: Standard deviation of 2 years around the expected initial selection period equal to the earlier of 20 years and attained age 85.

2. Grading-off period in years (immediately following the initial selection period): Standard deviation of two years around expected grade-off period of 10 years.

3. Ultimate mortality as a percentage of the mortality table: Standard deviation of 5% around 100% of the standard mortality assumption.

These three variables were modeled as mutually independent.

D. Other sources of volatility not included in this analysis

Although not modeled for this case study, it is worth considering how mortality rate volatility may arise from additional causes and how these can be evaluated.

1. Extreme long-term events: Outside of the trends and volatility of mortality improvement captured above, it is conceivable that events may cause mortality rates to change faster and more abruptly than anticipated in the baseline assumption, even after reflecting mortality improvement trend volatility that was derived from 1970 to 2010 historical levels. These changes could result in higher or lower experienced mortality. For example, a medical breakthrough can have a quick and long-term reduction on future death rates related to a specific condition or disease, and shift the mortality curves substantively from their current levels. Alternatively, a new drug-resistant bacterium or virus could cause immediate and long-term increases in deaths by infection.

2. Catastrophic short-term events: Unlike the extreme long-term events, some deviations in mortality trends may have a significant temporary impact before experience reverts to normal. Examples of these types of events include pandemics and acts of terrorism.

5 We believe that population mortality improvement is a reasonable source of data for developing volatility and correlation statistics for insured lives. However, insurers could also use their own data for developing these assumptions.

6 Actuaries should examine the underwriting process and mortality experience of the blocks of business when developing volatility parameters for this risk.

7 While this case study explicitly addresses underwriting risk for the life insurance block, companies have also experienced anti-selection from payout annuitants. The degree of this risk could be examined as basis risk or long-term underwriting risk.

These additional sources of volatility tend to have effects that move mortality in one direction, shifting the overall results. As such, modeling extreme events could produce asymmetrical results. We chose not to model the impact of these extreme events in this case study because we wanted to focus on symmetrical risk factors. We have demonstrated the effect of these extreme events in other papers and presentations.[8]

COST OF VOLATILITY

Figure 8 compares the baseline values in Figure 6 with the results of the 1,000 scenarios of projected mortality rates reflecting the sources of volatility described above, in which a pattern emerges from the stochastic simulations producing consistently lower mean and median distributable earnings than the baseline deterministic values.

This demonstrates that the stochastic mortality model has a small but measurable impact on average over many scenarios.

In other studies, we have found that when we perform stochastic analysis on assets with static assumptions for liability risk (e.g., mortality, longevity, and underwriting), the average overall scenarios will converge to the deterministic baseline. However, if dynamic assumptions for liability risks are used instead, the tail percentile values show an asymmetric dispersion, resulting in small but consistent divergence between the average of the stochastic scenarios and the deterministic baseline.

In those other studies and in this case study, the fact that economic liability under the dynamic assumptions is more than that under static assumptions is no coincidence but rather reflects the asymmetry in the payout patterns.

A possible explanation describing the phenomenon for the annuity line of business is as follows: While the average annuitant has an equal chance of living longer than expected or dying sooner than expected (with symmetric volatility assumptions), reflecting volatility increases the range of possible values—both increasing and decreasing values. However, this asymmetry stems from the fact that there is a limit to how much sooner an annuitant might die (i.e., on or after the valuation date), but the date to which they might survive is open-ended. Hence, the premature death can eliminate a limited number of annuity payments, but the unexpected survivor could receive a far greater number of additional payments. The degree that this asymmetry expresses itself in the present value of cash-flow statistics depends not only on the level of volatility but also the chosen discount rates.

For life insurance, death benefit payments are a leveraged cash flow net of premium and investment income. In aggregate, excess losses resulting from premature deaths in extremely adverse scenarios exceed interest gains and additional premiums from favorable scenarios with projected lower levels of mortality.

This average cost in excess of the baseline will not be reflected in the insurer's pricing unless a stochastic mortality projection is incorporated. An insurer investing its capital to issue life and annuity products accepts this risk and should be compensated for this additional cost.

FIGURE 8: COMPARISON OF BASELINE WITH STOCHASTIC MEAN RESULTS

PRESENT VALUE OF DISTRIBUTABLE EARNINGS

	COMBINED		ANNUITY		TERM	
DISCOUNT RATE	DETERMINISTIC BASELINE	STOCHASTIC MEAN	DETERMINISTIC BASELINE	STOCHASTIC MEAN	DETERMINISTIC BASELINE	STOCHASTIC MEAN
4%	4,022,238	3,882,026	145,925	139,145	3,876,313	3,742,881
8%	525,466	459,126	21,033	17,705	504,433	441,421
12%	(1,100,518)	(1,134,054)	(47,894)	(49,812)	(1,052,624)	(1,084,243)
IRR	9.00%	8.86%	9.00%	8.72%	9.00%	8.86%

8 Examples include:
Silverman, S. & D. Theodore, Considering the cost of longevity volatility on VA guaranteed living benefits. Accessed January 25, 2016, at http://us.milliman.com/uploadedFiles/Solutions/Products/cost-of-longevity-volatility.pdf.

Silverman, S. (June 29, 2011). Mortality improvement volatility and longevity risk: Implications for the US. Society of Actuaries webcast. Accessed January 25, 2016, at http://us.milliman.com/uploadedFiles/Solutions/Products/mortality-improvement-volatility-longevity.pdf.

Silverman, S. & P. Simpson. (October 2011). Case study: Modelling longevity risk for Solvency II. Milliman research report. Accessed January 25, 2016, at http://www.milliman.com/uploadedFiles/Solutions/Products/modelling-longevity-risk.pdf.

Silverman, S. (March-April 2010). Fatal factors: Managing mortality risk. Contingencies. Accessed January 25, 2016, at http://www.milliman.com/uploadedFiles/Solutions/Products/managing-mortality-risk.pdf.

Silverman, S. & D. Theodore. (Q4 2014) Evaluating the cost of longevity in variable annuity living benefits. Milliman Variable Annuity Market Update. Accessed January 25, 2016, at http://us.milliman.com/uploadedFiles/Solutions/Products/evaluating-cost-longevity.pdf.

Silverman, S. (May 2015). Variable annuity pricing considerations. Society of Actuaries Life & Annuity Symposium session. Accessed January 25, 2016, at https://www.soa.org/Files/Pd/2015/las/2015-las-session-70.pdf.

FIGURE 9: SUMMARY OF STOCHASTIC RESULTS – PAYOUT ANNUITY

			PRESENT VALUE OF DISTRIBUTABLE EARNINGS				
DISCOUNT RATE	DETERMINISTIC BASELINE	BASELINE WITH FIXED MARGIN	50TH PERCENTILE	75TH PERCENTILE	90TH PERCENTILE	95TH PERCENTILE	99TH PERCENTILE
4.00%	$145,925	$31,751	$140,853	$85,687	$37,066	$12,356	($79,758)
8.00%	$21,033	($46,545)	$18,051	($14,291)	($44,248)	($59,559)	($111,106)
12.00%	($47,894)	($92,432)	($49,289)	($72,684)	($91,336)	($102,393)	($133,179)
IRR	9.00%	5.36%	8.89%	7.30%	5.53%	4.55%	-1.59%

FIGURE 10: SUMMARY OF STOCHASTIC RESULTS – TERM INSURANCE

			PRESENT VALUE OF DISTRIBUTABLE EARNINGS				
DISCOUNT RATE	DETERMINISTIC BASELINE	BASELINE WITH FIXED MARGIN	50TH PERCENTILE	75TH PERCENTILE	90TH PERCENTILE	95TH PERCENTILE	99TH PERCENTILE
4.00%	$3,876,313	$2,061,961	$3,760,352	$3,180,835	$2,578,476	$2,153,651	$1,598,557
8.00%	$504,433	($490,955)	$443,339	$104,379	($229,090)	($488,176)	($817,206)
12.00%	($1,052,624)	($1,652,843)	($1,081,943)	($1,308,654)	($1,517,993)	($1,672,938)	($1,893,508)
IRR	9.00%	6.95%	8.88%	8.20%	7.53%	7.00%	6.26%

STOCHASTIC PROJECTIONS OF MORTALITY AND LONGEVITY

We performed stochastic projections reflecting the volatility from trend risk, basis risk, and long-term underwriting risk around the best estimate mortality assumptions. We generated 1,000 random stochastic mortality scenarios and in each scenario we calculated the distributable earnings for the term life insurance and payout annuity products. Figures 9 and 10 compare the percentile rankings separately for these blocks of business.

Note that the stochastic analysis starts with the baseline assumptions and then reflects volatility around the baseline. For the payout annuity, Figure 9 showed that the deterministic results with fixed margins produced results that were between the 90th and 95th percentile ranks. For term life, Figure 10 showed that the deterministic results with the fixed margins were close to the 95th percentile.

The deterministic scenario with a fixed margin is not directly comparable with the specific stochastic scenario corresponding to the same IRR. However, the stochastic analysis provides useful information and assists in putting that deterministic stress scenario in context. For example, if the intention of the deterministic projection with the fixed margin was to be a pricing scenario that attempted to be one standard deviation away from the mean, one may conclude that the margins chosen were too conservative. The stochastic analysis is a useful way to understand the appropriateness of fixed risk margins.

SAVINGS FROM DIVERSIFICATION

Our stochastic analysis reflects volatility generated in a manner that retains the historical level of correlation in changes in mortality rates across ages and gender. As such, we can use the analysis to examine the diversification benefits of having both life insurance and annuity blocks of business.

Importantly, when ranking scenarios, the combined financials of the insurance portfolio (i.e., of the term life insurance and the payout annuities) for percentile ranks greater than 50% are generally better than the sum of the separate financials for the two lines of business. Below we compare the results of the combined business to each block separately at selected percentiles.

FIGURE 11: SAVINGS FROM DIVERSIFICATION - MEDIAN

PRESENT VALUE OF DISTRIBUTABLE EARNINGS
SAVINGS (LOSS) FROM DIVERSIFICATION – 50TH PERCENTILE (MEDIAN)

DISCOUNT RATE	(A) COMBINED	(B) ANNUITY	(C) TERM	SAVINGS FROM DIVERSIFICATION (A)–[(B)+(C)]
4.00%	$3,883,071	$140,853	$3,760,352	($18,134)
8.00%	$469,296	$18,051	$443,339	$7,906
12.00%	($1,134,811)	($49,289)	($1,081,943)	($3,578)
IRR	8.89%	8.89%	8.88%	

FIGURE 12: SAVINGS FROM DIVERSIFICATION - 75TH PERCENTILE

PRESENT VALUE OF DISTRIBUTABLE EARNINGS
SAVINGS FROM DIVERSIFICATION – 75TH PERCENTILE

DISCOUNT RATE	(A) COMBINED	(B) ANNUITY	(C) TERM	SAVINGS FROM DIVERSIFICATION (A)–[(B)+(C)]
4%	$3,296,649	$85,687	$3,180,835	$30,127
8%	$112,191	($14,291)	$104,379	$22,102
12%	($1,358,610)	($72,684)	($1,308,654)	$22,729
IRR	8.21%	7.30%	8.20%	

FIGURE 13: SAVINGS FROM DIVERSIFICATION - 90TH PERCENTILE

PRESENT VALUE OF DISTRIBUTABLE EARNINGS
SAVINGS FROM DIVERSIFICATION – 90TH PERCENTILE

DISCOUNT RATE	(A) COMBINED	(B) ANNUITY	(C) TERM	SAVINGS FROM DIVERSIFICATION (A)–[(B)+(C)]
4.00%	$2,703,189	$37,066	$2,578,476	$87,646
8.00%	($219,509)	($44,248)	($229,090)	$53,829
12.00%	($1,564,205)	($91,336)	($1,517,993)	$45,125
IRR	7.56%	5.53%	7.53%	

FIGURE 14: SAVINGS FROM DIVERSIFICATION - 95TH PERCENTILE

PRESENT VALUE OF DISTRIBUTABLE EARNINGS
SAVINGS FROM DIVERSIFICATION – 95TH PERCENTILE

DISCOUNT RATE	(A) COMBINED	(B) ANNUITY	(C) TERM	SAVINGS FROM DIVERSIFICATION (A)–[(B)+(C)]
4.00%	$2,270,528	$12,356	$2,153,651	$104,521
8.00%	($494,029)	($59,559)	($488,176)	$53,706
12.00%	($1,750,889)	($102,393)	($1,672,938)	$24,442
IRR	7.01%	4.55%	7.00%	

FIGURE 15: SAVINGS FROM DIVERSIFICATION - 99TH PERCENTILE

PRESENT VALUE OF DISTRIBUTABLE EARNINGS
SAVINGS FROM DIVERSIFICATION – 99TH PERCENTILE

DISCOUNT RATE	(A) COMBINED	(B) ANNUITY	(C) TERM	SAVINGS FROM DIVERSIFICATION (A)–[(B)+(C)]
4.00%	$1,690,322	($79,758)	$1,598,557	$171,523
8.00%	($826,082)	($111,106)	($817,206)	$102,231
12.00%	($1,968,886)	($133,179)	($1,893,508)	$57,802
IRR	6.33%	-1.59%	6.26%	

While summing the median results from the separate life and annuity projections produces results similar to the median results on a combined basis, we see the diversification benefits emerge as we examine adverse scenarios. Given that we started with an insurance portfolio that has more life insurance exposure, the combined results are heavily influenced by the life insurance projections. However, a clear trend demonstrates the benefit of the annuity portfolio, mixed in with the life insurance portfolio, as we go deeper into the tail. As such, pricing objectives and economic capital may benefit from the recognition of stochastic analysis that allows for the quantification of these offsetting risks.

FIXED MARGIN VS. STOCHASTIC MODELING

As noted above, the fixed margin runs produced results that were between the 90th and 95th percentile scenarios for the payout annuity, and about the 95th percentile for the term life. Looking at the combined results, we see how reflecting the offsetting risks produces a different result:

The fixed margin for the combined results is now shown to be more conservative, falling between the 95th and 99th percentile scenarios.

FIGURE 16: SUMMARY OF STOCHASTIC RESULTS – COMBINED

PRESENT VALUE OF DISTRIBUTABLE EARNINGS

DISCOUNT RATE	DETERMINISTIC BASELINE	BASELINE WITH FIXED MARGIN	50TH PERCENTILE	75TH PERCENTILE	90TH PERCENTILE	95TH PERCENTILE	99TH PERCENTILE
4.00%	$4,022,238	$2,093,713	$3,883,071	$3,296,649	$2,703,189	$2,270,528	$1,690,322
8.00%	$525,466	($537,499)	$469,296	$112,191	($219,509)	($494,029)	($826,082)
12.00%	($1,100,518)	($1,745,275)	($1,134,811)	($1,358,610)	($1,564,205)	($1,750,889)	($1,968,886)
IRR	9.00%	6.89%	8.89%	8.21%	7.56%	7.01%	6.33%

Consider if the insurance company had set the fixed margin to address a risk tolerance at approximately the 90th percentile. While the selected fixed margin would have been a reasonable attempt to manage risk at that level for the term life and payout annuities separately, it did miss the mark on a separate basis, and it would be even that much more excessively conservative for the combined product portfolio. For the combined portfolio, the fixed margin may correspond to a risk level between the 95th and the 99th percentiles instead of the 90th percentile. This insurer with a risk tolerance nearer the 90th percentile could have used reduced fixed margins to achieve that goal. For example, in this case, the explicit margins applied to the mortality table and the annual mortality improvement could be revised as follows to produce a return more closely aligned with the 90th percentile:

a) Replaced 5.00% by 3.00% of annual mortality rates

b. Replaced 0.50% by 0.40% as the adjustment to annual mortality improvement

That is, for the term life insurance, the adjusted mortality will be:

a. 103% of best estimate annual mortality rates

b. Best estimate annual mortality improvement rates reduced 0.40%

For the payout annuity, the adjusted mortality will be:

a. 97% of best estimate annual mortality rates

b. Best estimate annual mortality improvement rates increased 0.40%

The resulting distributable earnings are summarized below:

FIGURE 17: DEVELOPING REVISED FIXED MARGIN

PRESENT VALUE OF DISTRIBUTABLE EARNINGS – REVISED FIXED MARGIN
3% MARGIN IN MORTALITY (LIFE = 103% Q_X / ANNUITY = 97% Q_X)
AND 0.40% MARGIN IN IMPROVEMENT (LIFE: -0.40% / ANNUITY +0.40%)

DISCOUNT RATE	PRESENT VALUE OF DISTRIBUTABLE EARNINGS			COST OF MARGIN VS. BASELINE
	COMBINED	ANNUITY	TERM	
4%	2,648,076	$66,043	$2,582,033	$1,374,162
8%	(223,971)	($25,704)	($198,266)	$749,436
12%	(1,550,119)	($78,345)	($1,471,774)	$449,601
IRR	7.55%	6.63%	7.58%	1.45%

It may be observed that the combined financial results are now reasonably close to the 90th percentile values from the stochastic projections.

FIGURE 18: COMPARISON OF RESULTS

PRESENT VALUE OF DISTRIBUTABLE EARNINGS
SUMMARY OF DETERMINISTIC AND STOCHASTIC RESULTS – COMBINED

DISCOUNT RATE	DETERMINISTIC BASELINE	BASELINE WITH REVISED FIXED MARGIN	90TH PERCENTILE
4.00%	$4,022,238	2,648,076	$2,703,189
8.00%	$525,466	(223,971)	($219,509)
12.00%	($1,100,518)	(1,550,119)	($1,564,205)
IRR	9.00%	7.55%	7.56%

It is also worth noting the significant savings in the cost of the revised margin over that of the original margin shown in Figure 7.

FIGURE 19: SAVINGS FROM REVISED MARGIN

PRESENT VALUE OF DISTRIBUTABLE EARNINGS
SUMMARY OF DETERMINISTIC AND STOCHASTIC RESULTS – COMBINED

DISCOUNT RATE	COST OF ORIGINAL FIXED MARGIN	COST OF REVISED FIXED MARGIN	SAVINGS FROM REVISED MARGIN
4.00%	$1,928,525	$1,374,162	$554,363
8.00%	$1,062,965	$749,436	$313,529
12.00%	$644,757	$449,601	$195,156
IRR	2.11%	1.45%	0.66%

Therefore, in this example, the use of the reduced margins has the direct result of allowing for more competitive products while satisfying the desired risk tolerance of the insurer. Alternatively, this approach may assist in recognizing redundancies in economic capital calculations.

MANAGING BUSINESS MIX REFLECTING DESIRED RISK TOLERANCE

A major component of a company's risk management is the decision to grow/acquire/divest different types of business. However, without analyzing the direct risk interaction of the different types of business, these companies may be making decisions without important information. For example, if a type of business is significantly diversifying from the company's existing portfolio, it may be able to offer more competitive pricing. On the other hand, understanding the interaction of these types of business may be useful in determining if a block of business should be divested.

In this section of the paper, we explore the relationship of different mixes of term life and payout annuities. By varying the volume of the payout annuity business relative to a fixed amount of term life in force, we can draw some inferences about how a company may opt to encourage the sales of certain products or acquire blocks of business as a form of risk management.

FIGURE 20: TERM/ANNUITY MIX – BASELINE DETERMINISTIC PROJECTION ("BEST ESTIMATE")

SENSITIVITY TO DIFFERENT LEVELS OF ANNUITIES RELATIVE TO TERM

	PRESENT VALUE OF DISTRIBUTABLE EARNINGS						
DISCOUNT RATE	100% TERM	100% ANNUITY	500% ANNUITY	1000% ANNUITY	100% TERM + 100% ANNUITY	100% TERM + 500% ANNUITY	100% TERM + 1000% ANNUITY
4%	3,876,313	145,925	729,626	1,459,252	4,022,238	4,605,939	5,335,565
8%	504,433	21,033	105,164	210,327	525,466	609,597	714,760
12%	(1,052,624)	(47,894)	(239,472)	(478,944)	(1,100,518)	(1,292,096)	(1,531,567)
IRR	9.00%	9.00%	9.00%	9.00%	9.00%	9.00%	9.00%

FIGURE 21: TERM/ANNUITY MIX – ORIGINAL FIXED MARGIN

SENSITIVITY TO DIFFERENT LEVELS OF ANNUITIES RELATIVE TO TERM – ORIGINAL FIXED MARGIN
5% MARGIN IN MORTALITY (LIFE = 105% Q_X / ANNUITY = 95% Q_X)
AND 0.50% MARGIN IN IMPROVEMENT (LIFE: -0.50% / ANNUITY +0.50%)

	PRESENT VALUE OF DISTRIBUTABLE EARNINGS - ORIGINAL FIXED MARGIN			COST OF ORIGINAL FIXED MARGIN (BASELINE LESS FIXED MARGIN)		
DISCOUNT RATE	100% TERM + 100% ANNUITY	100% TERM + 500% ANNUITY	100% TERM + 1000% ANNUITY	100% TERM + 100% ANNUITY	100% TERM + 500% ANNUITY	100% TERM + 1000% ANNUITY
4%	2,093,713	2,220,718	2,379,475	1,928,525	2,385,221	2,956,089
8%	(537,499)	(723,677)	(956,400)	1,062,965	1,333,274	1,671,160
12%	(1,745,275)	(2,115,003)	(2,577,164)	644,757	822,908	1,045,597
IRR	6.89%	6.70%	6.53%	2.11%	2.29%	2.47%

As can be seen in Figure 20, using deterministic pricing is not particularly helpful when considering business management. If each existing business is priced to earn an IRR of 9%, layering on additional business that earns that same return will not have a material impact on profitability.

When utilizing a deterministic valuation methodology, fixed margins may price for risk, but they do not assist in understanding the risk level of the combined business. In Figure 21 we demonstrate how using the fixed margin approach only shows more cost as more annuity business is layered on.

Essentially, when using the deterministic approach, the overall return is a weighted average of the returns of the business included, without reflecting any diversification benefits. This approach is not so helpful when considering business management.

Now let us consider how the different levels of annuity business mixed in with the life insurance portfolio fare at different percentile values using our stochastic analysis.

FIGURE 22 - 24: TERM/ANNUITY MIX – SUMMARY OF RESULTS

SENSITIVITY TO DIFFERENT LEVELS OF ANNUITIES RELATIVE TO TERM - DETERMINISTIC VS. FIXED MARGIN VS. STOCHASTIC
ORIGINAL FIXED MARGIN: 5% MARGIN IN MORTALITY (LIFE = 105% Q_X / ANNUITY = 95% Q_X) AND 0.50% MARGIN IN IMPROVEMENT (LIFE: -0.50% / ANNUITY +0.50%)
REVISED FIXED MARGIN: 3% MARGIN IN MORTALITY (LIFE = 103% Q_X / ANNUITY = 97% Q_X) AND 0.40% MARGIN IN IMPROVEMENT (LIFE: -0.40% / ANNUITY +0.40%)

FIGURE 22: 100% BASELINE TERM AND 100% BASELINE ANNUITIES

DISCOUNT RATE	DETERMINISTIC BASELINE	BASELINE WITH ORIGINAL FIXED MARGIN	BASELINE WITH REVISED FIXED MARGIN	75TH PERCENTILE	90TH PERCENTILE	95TH PERCENTILE	99TH PERCENTILE
4.00%	$4,022,238	$2,093,713	$2,648,076	$3,296,649	$2,703,189	$2,270,528	$1,690,322
8.00%	$525,466	($537,499)	($223,971)	$112,191	($219,509)	($494,029)	($826,082)
12.00%	($1,100,518)	($1,745,275)	($1,550,119)	($1,358,610)	($1,564,205)	($1,750,889)	($1,968,886)
IRR	9.00%	6.89%	7.55%	8.21%	7.56%	7.01%	6.33%

Note that the target IRR for the insurer has been previously determined to be 7.56% at the 90th percentile.

FIGURE 23: 100% BASELINE TERM AND 500% BASELINE ANNUITIES

DISCOUNT RATE	DETERMINISTIC BASELINE	BASELINE WITH ORIGINAL FIXED MARGIN	BASELINE WITH REVISED FIXED MARGIN	75TH PERCENTILE	90TH PERCENTILE	95TH PERCENTILE	99TH PERCENTILE
4.00%	$4,605,939	$2,220,718	$2,912,247	$3,724,007	$3,080,165	$2,595,670	$1,885,676
8.00%	$609,597	($723,677)	($326,788)	$92,538	($253,078)	($570,978)	($945,061)
12.00%	($1,292,096)	($2,115,003)	($1,863,500)	($1,626,986)	($1,842,982)	($2,051,539)	($2,315,807)
IRR	9.00%	6.70%	7.43%	8.15%	7.57%	7.02%	6.31%

FIGURE 24: 100% BASELINE TERM AND 1000% BASELINE ANNUITIES

DISCOUNT RATE	DETERMINISTIC BASELINE	BASELINE WITH ORIGINAL FIXED MARGIN	BASELINE WITH REVISED FIXED MARGIN	75TH PERCENTILE	90TH PERCENTILE	95TH PERCENTILE	99TH PERCENTILE
4.00%	$5,335,565	$2,379,475	$3,242,461	$4,188,514	$3,393,817	$2,867,311	$2,129,869
8.00%	$714,760	($956,400)	($455,310)	$57,330	($413,059)	($723,879)	($1,204,147)
12.00%	($1,531,567)	($2,577,164)	($2,255,226)	($1,949,588)	($2,259,143)	($2,465,007)	($2,774,249)
IRR	9.00%	6.53%	7.32%	8.08%	7.40%	6.92%	6.20%

When more annuity business is added to the life insurance portfolio, the fixed margins become less in line with the insurer's 90th percentile risk target. For example, the fixed margin that was solved for to approximate the effect of the 90th percentile scenario under the original mix of life insurance and annuity business produces a result that is now between the 90th and 95th percentiles when the annuity component of the mix is larger.

It is worth noting that, unlike the fixed margin approach that shows a lower overall yield, the IRR of the 90th percentile rises slightly from 7.56% to 7.57% as the annuity sales increase fivefold. This would imply that the insurer could quintuple the amount of annuity business (compared with the original portfolio) and still achieve the same risk objective. However, we see deterioration of that risk objective as we move from five times the original size of the annuity business to 10 times. One may observe from this analysis that a fixed margin appropriate for one mix of business may not be appropriate for another mix of business.

CONCLUSION

This case study demonstrates that there is real value for an insurer in performing stochastic modeling with volatile mortality assumptions when pricing, setting deterministic margins, determining economic capital, and determining its optimal mix of business.

Even if companies are using stochastic asset analysis, the use of static liability margins may result in either understating or overstating some modeled risk. The excess levels will depend on the static margins currently being used and the liability volatility parameters chosen. We believe it is in companies' best interests to understand how their life insurance and annuity businesses may interact.

While this paper has focused on the diversification of mortality from life and annuity products, we also recognize that there are diversification benefits between mortality and investment risk. While the detailed analysis of this relationship is beyond the scope of this paper, we have demonstrated its effect in other papers and presentations.[9]

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9 See:
Silverman & Theodore, Considering the cost of longevity volatility on VA guaranteed living benefits.
Silverman & Theodore, Evaluating the cost of longevity in variable annuity living benefits.
Silverman, Variable annuity pricing considerations.